In a continuous, differentiable interval on a graph, if the start and end have the same codomain () value, how can we be sure that there exists a point in-between with a slope of zero? Central to many theorems in calculus is the one which proves this, the *mean value theorem*. This section will discuss *Rolle’s Theorem* and use it to prove the mean value theorem.

# Rolle’s Theorem

Rolle’s Theorem (theorem) – a theorem that says that between two points and in a function , under certain conditions there definitely is a point between and where .

## What Rolle’s Theorem Says

Figure

For a function and numbers in the domain and (Figure 1), if

1. is continuous on .
2. is differentiable on .

Then a number exists between and such that .

## Why Rolle’s Theorem is True

A function that fits within Rolle’s Theorem is in one of three cases:

|  |  |  |
| --- | --- | --- |
| (k is constant) | (for some x) | (for some x) |
|  |  |  |
| for all , so can be anywhere | By the Extreme Value Theorem (section 4.1), there is an extremum at in the interval. By Fermat’s Theorem (section 4.1), . | |

## How to Use Rolle’s Theorem

**Proof by Contradiction**: Every logical argument is either true or false. To prove which, try to find any case of all possible scenarios where something must be true. If there are none, it must be false.

To use Rolle’s Theorem to prove that a number exists between and in function where ,

1. Take the derivative of , .
2. Find a value of where .
3. If there is a solution of , the number . If there isn’t a solution of , then there is no value between and in where the slope is , so Rolle’s Theorem cannot be applied.

# The Mean Value Theorem

The Mean Value Theorem (theorem) – if certain conditions are met for function between points and , then there is some point where the slope of the line between and equals .

## What the Mean Value Theorem Says

Figure

For a function and numbers and , if

* is continuous on .
* is differentiable on .

Then there is a number such that

## Why the Mean Value Theorem is True

If a function is continuous on interval and differentiable on interval , then

1. Define the secant line as a function. and .

**Remember**, functions of a constant such as are also constants, so their derivatives are .

1. Define to be the difference between and the secant line at . .
2. Therefore,
3. Because is continuous on and differentiable on , then is continuous on and differentiable on . Because and the last sentence, is usable by Rolle’s Theorem.
4. Because satisfies Rolle’s Theorem, there is some number such that the derivative .
5. Because , . To solve for , take the inverse of .

## What the Mean Value Theorem Implies

* If a derivative for all in , then is constant in because it does not change in that domain.
* If there are two functions and and for all in , then is constant in , because and change with the same rate of change.

# How Would You Answer?

* What is Rolle’s Theorem? How do you know when it can be applied? Why is it true?
* What is the mean value theorem? When can it be applied? Why is it true? What does it imply?
* What do these theorems tell you about the velocity of a paper airplane flying in a parabola?